Making a Case for Modelling in Mathematics

By **Marc North**

Revisiting the mathematics-for-all debate

The Prime Minister's recent calls for compulsory mathematics study for all students to age 18-years (Sunak, 2023) has resurrected debates on whether and what mathematics students should be given access to at different stages of their schooling careers. Given the large number of students failing Mathematics at GCSE level and subsequently in GCSE-resits, not to mention the significant anxiety already experienced by so many students who have to continue with Mathematics to age 16, there is a case to be made for doing things differently. If compulsory mathematics-for-all is to become a reality, what may make mathematics more accessible and applicable is a different type of mathematics qualification that does not foreground the same type of abstract mathematical contents, largely disconnected from the students' daily lives, that characterises much of the current curriculum.

One alternative would be to prioritise a qualification that is less focused on the learning of abstract mathematical content and more focused on empowering students to use mathematics to support more enhanced participation in both their current and future lives – motivated, perhaps, by a critical citizenship agenda. This would involve more than just applying mathematics to a limited selection of real-life contexts, which often ends up with students using mathematics in overly-simplified pseudo real-life scenarios. Instead, this would be about equipping students with the skills needed to model and solve complex and intricate problems encountered in both familiar and unfamiliar authentic real-life and workplace contexts by drawing on a variety of mathematical, technological, contextual and other knowledge and tools that enable empowered decision-making. Arguably, this would ensure that students leave school better prepared to make informed and empowered decisions in their current and future personal lives, while at the same time serving the needs of employers seeking workers able to engage flexibly in complex problem-solving activities. This would distinguish the qualification from both Functional Mathematics and Core Mathematics, each of which are largely organised around mathematical content domains and underpinned by an agenda for mathematical competence (and, so, remain open to the criticism that the lower level of mathematical demand positions these qualifications as a sub-set or lower form of scientific Mathematics). This could also alleviate some of the anxiety and challenges experienced by so many students associated with the learning of abstract mathematics.

A broader definition of problem-solving

A key element of what I am arguing above is a broader definition of problem-solving than might currently be operationalised in the curriculum, classrooms and assessments, and also a specific emphasis on the importance of *modelling* for facilitating empowered decision-making. In current Primary and Secondary mathematics practices, 'problem-solving' loosely translates to solving mathematical problems that are couched in pseudo-real contextual problems. Although there may be more than one way to solve the problems, it is usually the most efficient mathematical method that is prioritised and privileged. And, success in solving these problems is based on being able to work mathematically and in a mathematically-accurate and appropriate way, hereby showcasing an understanding of a particular mathematical concept or procedure. The contexts are a means-to-an-end to support the application of mathematics; understanding of the context is not a pre-requisite for successful engagement with the problem, and for some students the contexts actually get in the way of them accessing the mathematical elements. By successfully completing these problem-solving activities, students showcase their mathematical skills but don't develop an enhanced understanding of the context.

Contrast this with the broader definition of problem-solving captured in Fig. 1. Here problem-solving involves the capacity to draw on a variety of knowledge and tools (contextual, mathematical, technological) to *model* possible solutions and possible alternative ways of working to inform decision-making. 'Modelling' here is understood to represent the process whereby a specific event or component of a real-world problem ('focal event') based in a genuine and authentic 'contextual environment' is reformulated according to mathematical principles and structures ('mathematical model') and investigated via mathematical and technological methods to better understand the focal event. This problem-solving approach recognises the crucial roles of a combination of contextual knowledge, basic mathematical competency, and technological skills for facilitating critical engagement

with daily life and workplace problem-solving experiences. However, this approach also recognises that a 'mathematised' view of the world represents a limited view and that the principles that inform how people act and the decisions that they make in real-life settings commonly don't reflect heavily mathematical ways of working – which is why interpreting mathematical results to determine their validity and suitability in a particular contextual focal event (processes 5 and 6) is an essential part of the modelling process. That said, this approach privileges the view that being able to model in this way is empowering because it enables an individual to explore how someone might think, act, behave and decide differently from a more mathematically-informed perspective.

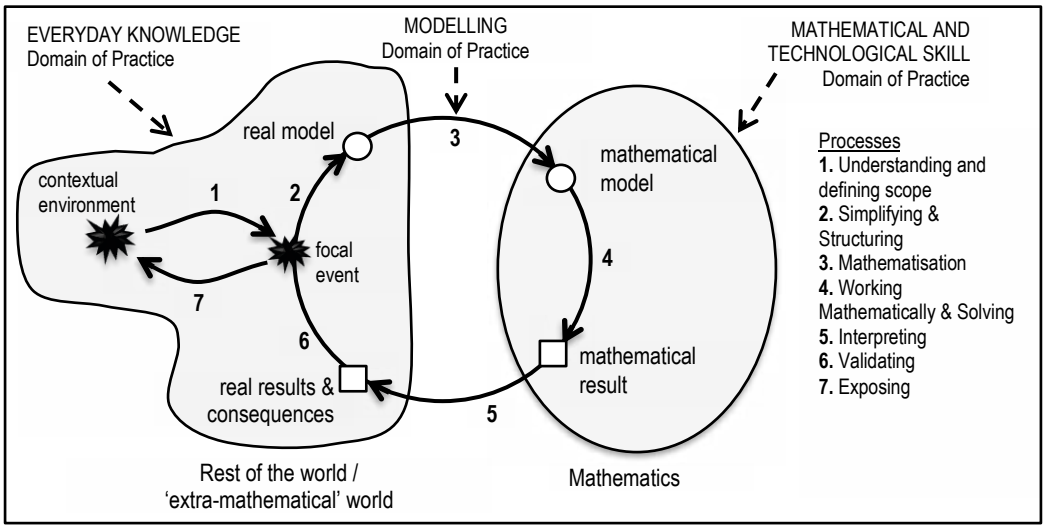


Figure 1: Problem-solving Cycle – adapted from (Blum and Ferri, 2009)

The dreaded remortgaging deadline!

My aim below is to exemplify how we might engage in such modelling activity in daily-life settings and to show how having the capacity to do this facilitates opportunities for empowered decision-making. The example I use arose when, quite recently, I had to go through the onerous task of remortgaging at the end of a fixed-term deal. What makes this real-life problem-solving task complicated is the large array of options to consider, each with subtly different conditions, criteria, costs and variables (see Fig. 2)

To make the decision-making process more manageable for myself, I pick a few options that meet my immediate needs (e.g. wanting another 2-year fixed-rate deal) and then used a spreadsheet to create a model (below) showing how the value of the loan will change over the 2-year period under the different conditions on offer (see Fig. 3).

There are a number of considerations I want to draw out from this problem-solving-with-modelling episode.

1. Elementary mathematics ...

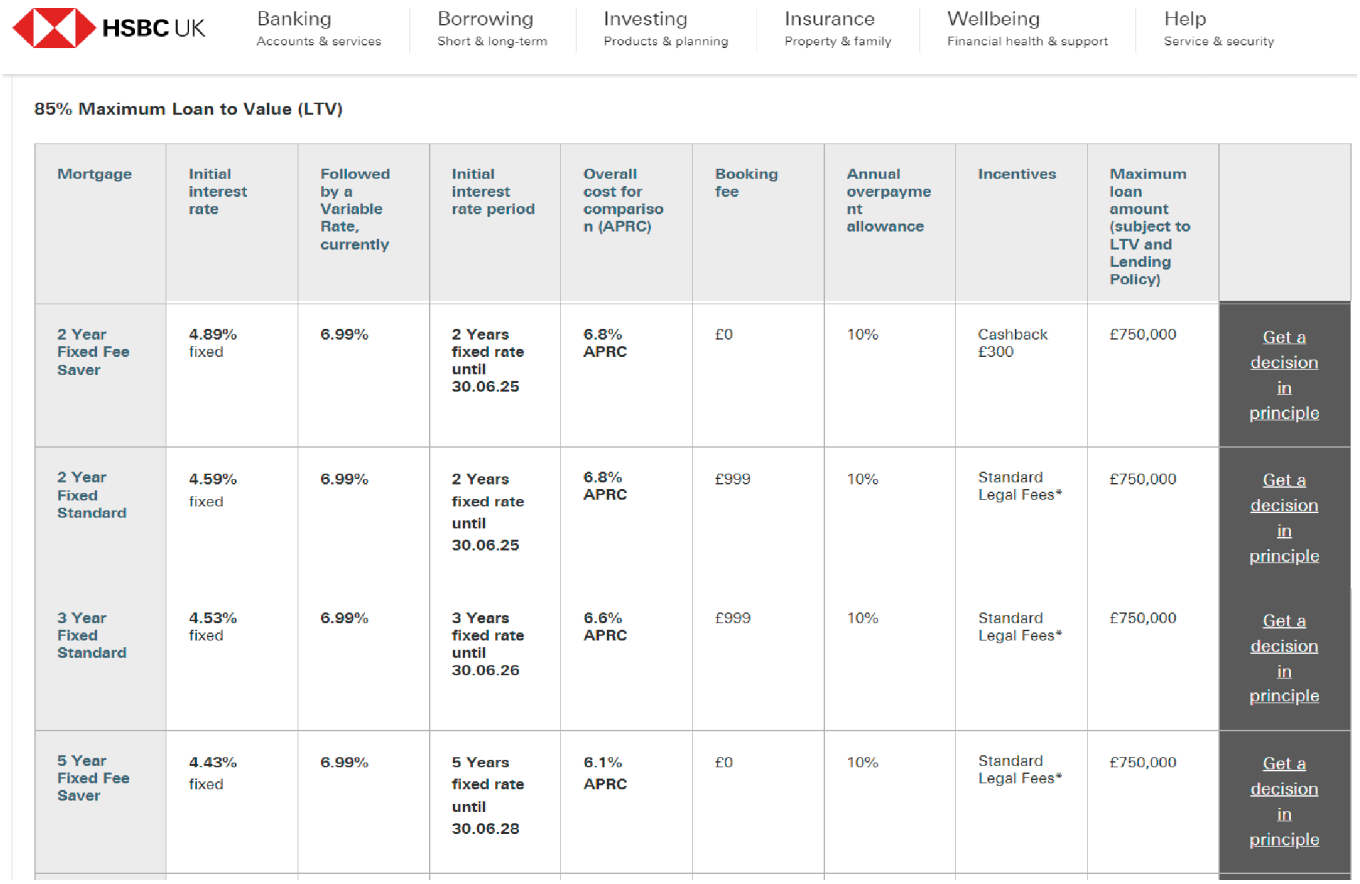
In most real-life problem-solving experiences, the mathematics needed is fairly elementary and commonly involves some or other combination of arithmetic calculations, proportional reasoning, and measures. For the most part, we don't need understanding of sophisticated abstract mathematical concepts to engage with real-world problem-solving experiences, and for those small number of instances where this may be necessary, there is usually a technological tool or resource that will support with this. What is more important is being able to identify what mathematics may be necessary, how to use available tools, and particularly how to make sense of any results.

2. ... but complex reasoning

Despite the fairly elementary level of mathematics, constructing a useful model that represents the problem in a sufficiently accurate way requires complex critical reasoning. Decisions have to be made about what information should be included and what information can be ignored, how to structure the model, what calculations to use, and whether the results are fit-for-purpose for the context. High-level reasoning is also required to understand the impact of changes in different variables on the problem outcome. Interpretation is a further essential component of the modelling process and the user of the model has to be able to analyse and make sense of the model to support their decision-making. Sometimes it is also important to be able to communicate and justify findings and decisions in an appropriate and accessible way to different audiences, drawing on a range of tools (e.g. pictures, graphs, calculations) to do this.

3. Real-world problem-solving involves real-life data and artefacts

Real-life data and artefacts contain messy numbers, complex contextual knowledge, and sometimes



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85% Maximum Loan to Value (LTV)

Mortgage	Initial interest rate	Followed by a Variable Rate, currently	Initial interest rate period	Overall cost for comparison (APRC)	Booking fee	Annual overpayment allowance	Incentives	Maximum loan amount (subject to LTV and Lending Policy)	
2 Year Fixed Fee Saver	4.89% fixed	6.99%	2 Years fixed rate until 30.06.25	6.8% APRC	£0	10%	Cashback £300	£750,000	Get a decision in principle
2 Year Fixed Standard	4.59% fixed	6.99%	2 Years fixed rate until 30.06.25	6.8% APRC	£999	10%	Standard Legal Fees*	£750,000	Get a decision in principle
3 Year Fixed Standard	4.53% fixed	6.99%	3 Years fixed rate until 30.06.26	6.6% APRC	£999	10%	Standard Legal Fees*	£750,000	Get a decision in principle
5 Year Fixed Fee Saver	4.43% fixed	6.99%	5 Years fixed rate until 30.06.28	6.1% APRC	£0	10%	Standard Legal Fees*	£750,000	Get a decision in principle

Figure 2: HSBC Mortgage Rates, 28 April 2023, as given at www.hsbc.co.uk/mortgages/move-your-mortgage/rates/#fixed-rate. (It is not my intention to promote this particular bank. It's a platform I'm familiar with and the way they present their mortgage information was helpful to support this discussion.)

complex contextual language and terminology. Understanding of the contextual elements is an essential element in the problem-solving process and without this understanding it is not possible to effectively solve problems in these contexts or to construct helpful models to support decision-making.

4. Models provide a limited but helpful perspective of reality Models provide a simplified and limited representation of reality

It is important to recognise that as helpful as models can be for supporting decision-making, the models we create will always provide a simplified and limited representation of reality. This is because it is impossible to consider and understand all of the contextual elements in a contextual environment or focal event, and the models we construct can't possibly capture all of the nuances of the contextual scenario – after all, in the case of mortgages, there are entire degree qualifications and departments devoted to generating models of different financial scenarios. As such, all we are aiming for is to be able to generate models that contain enough detail to enable us to make informed choices – in other words, to make them fit-for-our-purposes. This means that sometimes we will make do with simple calculations and models and other times we might opt for more complex models to give us more insight. For example, for the remortgage problem, instead of developing a spreadsheet model showing how each loan changes over the 2-year period, we could have just and only multiplied the monthly repayment value for each option by 24 to compare the total repayment costs. This would have given us a basic sense of which option would involve a higher repayment amount which, depending on our needs, may have been enough information to support our decision-making. But, this approach does only provide limited information, and the more detailed spreadsheet model makes it possible to better understand and investigate a wider range of considerations – such as, what would happen if we make a lump-sum payment into the loan halfway through the loan period; or the impact of an interest rate change; and so on. Being able to develop more sophisticated models, then, has the potential to support more informed and empowered decision-making, a point I argue more in (7) below.

5. Calculations are in service to a larger goal for task completion and/or decision-making

In real-life problem-solving experiences, the mathematical calculations are largely in service to a larger goal for decision-making, task completion, or sense-making of a contextual situation; and, often,

2-Year Fixed-Term Offer #1 - Fixed Fee Saver 4.89%

Remortgage Details			
Property Value	£350,000	Interest rate	4.89% per year
Remortgage amount	£250,000	Monthly Repayment	£1,445.00
Total Mortgage Length	25 years	Booking fee	£0.00
		Legal Fee	£295.00
		Cashback	£300.00

Month	Opening	Interest	Balance with interest	Monthly Repayment	Closing balance
Month 1	£250,000.00	£1,018.75	£251,018.75	£1,445.00	£249,573.75
Month 2	£249,573.75	£1,017.01	£250,590.76	£1,445.00	£249,145.76
Month 3	£249,145.76	£1,015.27	£250,161.03	£1,445.00	£248,716.03
Month 4	£248,716.03	£1,013.52	£249,729.55	£1,445.00	£248,284.55
Month 5	£248,284.55	£1,011.76	£249,296.31	£1,445.00	£247,851.31
Month 6	£247,851.31	£1,009.99	£248,861.30	£1,445.00	£247,416.30
Month 7	£247,416.30	£1,008.22	£248,424.52	£1,445.00	£246,979.52
Month 8	£246,979.52	£1,006.44	£247,985.97	£1,445.00	£246,540.97
Month 9	£246,540.97	£1,004.65	£247,545.62	£1,445.00	£246,100.62
Month 10	£246,100.62	£1,002.86	£247,103.48	£1,445.00	£245,658.48
Month 11	£245,658.48	£1,001.06	£246,659.54	£1,445.00	£245,214.54
Month 12	£245,214.54	£999.25	£246,213.79	£1,445.00	£244,768.79
Month 13	£244,768.79	£997.43	£245,766.22	£1,445.00	£244,321.22
Month 14	£244,321.22	£995.61	£245,316.83	£1,445.00	£243,871.83
Month 15	£243,871.83	£993.78	£244,865.61	£1,445.00	£243,420.61
Month 16	£243,420.61	£991.94	£244,412.55	£1,445.00	£242,967.55
Month 17	£242,967.55	£990.09	£243,957.64	£1,445.00	£242,512.64
Month 18	£242,512.64	£988.24	£243,500.88	£1,445.00	£242,055.88
Month 19	£242,055.88	£986.38	£243,042.26	£1,445.00	£241,597.26
Month 20	£241,597.26	£984.51	£242,581.77	£1,445.00	£241,136.77
Month 21	£241,136.77	£982.63	£242,119.40	£1,445.00	£240,674.40
Month 22	£240,674.40	£980.75	£241,655.15	£1,445.00	£240,210.15
Month 23	£240,210.15	£978.86	£241,189.00	£1,445.00	£239,744.00
Month 24	£239,744.00	£976.96	£240,720.96	£1,445.00	£239,275.96

TOTAL PAID (Repayments + Fees - Cashback)	£34,675.00
AMOUNT PAID OFF (Opening - Closing Balance)	£10,724.04
% OF ORIGINAL LOAN PAID OFF	4.3%

2-Year Fixed-Term Offer #2 - Fixed Standard 4.59%

Remortgage Details			
Property Value	£350,000	Interest rate	4.59% per year
Remortgage amount	£250,000	Monthly Repayment	£1,402.00
Total Mortgage Length	25 years	Booking fee	£999.00
		Legal Fee	£0.00
		Cashback	£0.00

Month	Opening	Interest	Balance with interest	Monthly Repayment	Closing balance
Month 1	£250,000.00	£956.25	£250,956.25	£1,402.00	£249,554.25
Month 2	£249,554.25	£954.55	£250,508.80	£1,402.00	£249,106.80
Month 3	£249,106.80	£952.83	£250,059.63	£1,402.00	£248,657.63
Month 4	£248,657.63	£951.12	£249,608.74	£1,402.00	£248,206.74
Month 5	£248,206.74	£949.39	£249,156.13	£1,402.00	£247,754.13
Month 6	£247,754.13	£947.66	£248,701.79	£1,402.00	£247,299.79
Month 7	£247,299.79	£945.92	£248,245.72	£1,402.00	£246,843.72
Month 8	£246,843.72	£944.18	£247,787.89	£1,402.00	£246,385.89
Month 9	£246,385.89	£942.43	£247,328.32	£1,402.00	£245,926.32
Month 10	£245,926.32	£940.67	£246,866.99	£1,402.00	£245,464.99
Month 11	£245,464.99	£938.90	£246,403.89	£1,402.00	£245,001.89
Month 12	£245,001.89	£937.13	£245,939.02	£1,402.00	£244,537.02
Month 13	£244,537.02	£935.35	£245,472.38	£1,402.00	£244,070.38
Month 14	£244,070.38	£933.57	£245,003.95	£1,402.00	£243,601.95
Month 15	£243,601.95	£931.78	£244,533.72	£1,402.00	£243,131.72
Month 16	£243,131.72	£929.98	£244,061.70	£1,402.00	£242,659.70
Month 17	£242,659.70	£928.17	£243,587.88	£1,402.00	£242,185.88
Month 18	£242,185.88	£926.36	£243,112.24	£1,402.00	£241,710.24
Month 19	£241,710.24	£924.54	£242,634.78	£1,402.00	£241,232.78
Month 20	£241,232.78	£922.72	£242,155.49	£1,402.00	£240,753.49
Month 21	£240,753.49	£920.88	£241,674.38	£1,402.00	£240,272.38
Month 22	£240,272.38	£919.04	£241,191.42	£1,402.00	£239,789.42
Month 23	£239,789.42	£917.19	£240,706.61	£1,402.00	£239,304.61
Month 24	£239,304.61	£915.34	£240,219.95	£1,402.00	£238,817.95

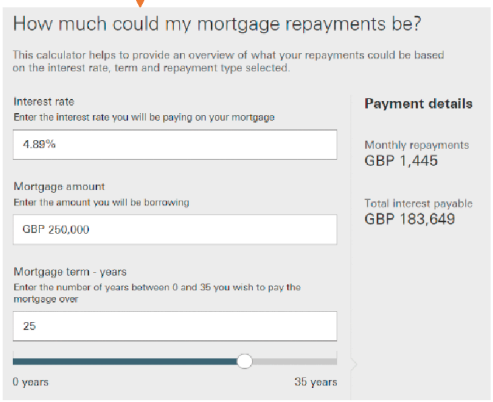
TOTAL PAID (Repayments + Fees - Cashback)	£34,647.00
AMOUNT PAID OFF (Opening - Closing Balance)	£11,182.05
% OF ORIGINAL LOAN PAID OFF	4.5%

Month	Opening	Interest	Balance with interest	Monthly Repayment	Closing balance
Month 1	£250,000.00	£1,018.75	£251,018.75	£1,445.00	£249,573.75
Month 2	£249,573.75	£1,017.01	£250,590.76	£1,445.00	£249,145.76

Interest
= Monthly interest rate × Opening balance
= (4.89% ÷ 12) × £249,573.75
= 0.4075% × £249,573.75
= 0.004075 × £249,573.75

Balance with interest
= Opening balance + Interest
= £249,573.75 + £1,017.01

Closing balance
= Balance with interest - Repayment
= £250,590.76 - £1,445.00



The original remortgage offers didn't include an indication of the Monthly Repayment amounts, so I had to use an online mortgage repayment calculator (alongside) to help with this (Source: HSBC Mortgage Calculator, accessed 28 April 2023 **https://www.hsbc.co.uk/mortgages/repayment calculator/**)

Figure 3: Spreadsheet model of two of the remortgage options

real-life considerations take preference over more mathematically-valid solutions. For the remortgaging problem, any mathematics used is in service to the broader goal of trying to decide which loan option best fits my personal current financial circumstances. This means that sometimes the 'best' mathematical solution does not always equate to the option that provides the most suitable choice for a given real-life experience – which is why interpreting and validating solutions are so important in a modelling process. For example, according to the remortgage models above, the second option with the lower interest rate (4.59%) is the better deal – not by much in terms of the Total Paid over the 2-years (£28.00), but more so in terms of the Amount Paid Off (£458.01 more) when comparing the starting balance to the final closing balance. However, this option also comes with a £999 upfront cost, and if I don't have these upfront funds available, then I may have to opt for the more expensive option.

6. **Technology is an essential component of the problem-solving and modelling process**

Since real-life data is messy, we don't expect to have to make sense of this data without helpful technologies such as calculators, spreadsheets, or online applications. So, in the modelling process, technological knowledge is equally as important as contextual knowledge and mathematical knowledge. In addition, having technological skills to inform and support our problem-solving activities means that we are able to consider a wider range of scenarios and to work with more complex information, all of which support the potential for more empowered decision-making.

7. **Modelling supports empowered decision-making**

I genuinely believe that being able to model in this way makes it possible to make more informed and empowered decisions, which could explain why these sorts of in-depth problem-solving and critical reasoning skills are so valued by employers. It is clearly possible to make choices in daily-life and workplace experiences without doing the sort modelling showcased above. However, having the skills and capacity to generate these sorts of models provides opportunities to explore and understand problems more fully and, so, to be able to make informed decisions rather than having to rely on others to support or influence our decision-making. Generating models also makes it possible to experiment and to notice and have discussions about things we may not have seen otherwise, such as the impact of changes in the interest rate, paying more than is necessary each month, or making a large lump-sum repayment.

Challenges with modelling – is it really worth it?

A focus on the type of problem-solving and modelling approach showcased above is not without challenges. First is the issue of which contexts are worthy of investigation and whether those contexts are of interest and relevance to the students, particularly when contexts link to scenarios that students may only encounter in future life and work experiences, such as mortgages or tax. Second is the increased language and comprehension demand that occurs when contextual elements are foregrounded, and (third) the need for both teachers and students to develop more detailed contextual knowledge about sophisticated real-life concepts. Fourth is the question of whether we actually need these types of modelling skills to be able to function effectively in real-life since, after all, people tend to get by just fine without an emphasis on modelling in the current curriculum. Fifth is the challenge of how this approach can be assessed when part of a school-qualification – for example, to ensure that familiarity or unfamiliarity with a context doesn't influence whether or not a student can successfully engage with a problem-solving scenario in an assessment setting.

All of these concerns are valid and speak to the complexity of taking seriously attempts to integrate authentic real-world problem-solving opportunities into academic learning experiences. However, we have an opportunity to rethink the type of engagement with mathematics we want students to have, particularly students who find mathematics difficult and may have had negative or unsuccessful mathematical learning experiences. And, doing more of the same kind of mathematics isn't going to make things better for them. Instead, by exposing students to a broader range of problem-solving experiences that involve modelling, and by giving equal consideration to the development of contextual, mathematical and contextual knowledge and skills, we create the potential for students to be better prepared for the world beyond school and empower them to be more independent self-managing citizens. Perhaps this goal alone makes any challenges with prioritising contextual problem-solving-with-modelling worth it?

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